ESTIMATION OF SIGNAL DEPENDENT NOISE PARAMETERS FROM A SINGLE IMAGE

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ABSTRACT

The additive white Gaussian noise (AWGN) is usually assumed in many image processing algorithms. However, these algorithms cannot effectively deal with the noise from actual cameras which is better modeled as signal dependent noise (SDN). In this paper, we focus on the SDN model and propose an algorithm to accurately estimate its parameters without any assumption of the noise types. The noise parameters are estimated by using the selected weak textured patches from a single noisy image. Experiments on synthetic noisy images are conducted to test the algorithm, which show that our noise parameter estimation outperforms the existing algorithms. And based on our estimation, the performance of image processing applications like Wiener filter can be effectively improved.

Index Terms— signal dependent noise model, noise measurement, homogeneous patches, PCA, denoising

1. INTRODUCTION AND RELATED WORK

Noise is one of the main factors to degrade image quality. Identifying the noise characteristics is of great importance for image processing applications such as denoising, edge detection image segmentation and so on. An additive white Gaussian noise (AWGN) is usually assumed in many noise level estimation algorithms [1, 2]. However, the noise of an actual camera is not AWGN and is better modeled as signal dependent noise (SDN) whose standard deviation is represented by a function of pixel intensity. The function which represents the standard deviation of the noise or the noise level is called the *noise level function* (NLF). It is desirable to build a practical algorithm to identify the NLF from a single image in a robust manner. The NLF is modeled with several parameters. Therefore the main goal of the SDN identification is to estimate these NLF parameters. Liu et al. [3] addressed the SDN estimation from a single image based on a piecewise smooth prior model and estimated an upper bound of the NLF by fitting the local mean and the local variance in each segment. In [4, 5, 6, 7], homogeneous patches are firstly detected and then the means and the variances of the detected patches are used to estimate the NLF parameters. However, these algorithms usually assume that at least one NLF parameter is known to simplify the estimation. For this assumption, these algorithms are not so practical.

The challenge in the SDN estimation is correct detection of the weak textured patches or the homogeneous patches. Once we have the weak textured patches, the NLF parameters can be estimated by the maximum likelihood (ML) estimator or other fitting algorithms. We have proposed a texture strength metric based on the gradient matrix of the image patch to detect the weak texture patches and a noise level estimation algorithm for the signal independent AWGN [2]. In this paper, we propose the NLF estimation algorithm for the SDN. The weak textured patches are selected by the same manner as [2]. The NLF parameters are estimated based on the means and the variances of the selected weak textured patches by the ML estimator. The experimental results demonstrate that our proposed algorithm works well for various scenes and outperforms the state-of-the-art algorithms.

2. PROPOSED SDN IDENTIFICATION

The main goal of the SDN estimation is to estimate the NLF parameters. First, we introduce a general signal dependent noise model with three parameters. Then, we propose a NLF parameter estimation algorithm.

2.1. Signal Dependent Noise Model

A general signal dependent noise model has been proposed to deal with different types of noise [8]. The observed noisy pixel value can be expressed by:

$$g = f + f^{\gamma} \cdot u + w, \tag{1}$$

where g is the noisy pixel value, f is the noise-free pixel value, γ is the exponential parameter, and u and w are zero-mean random variables with variances σ_u^2 and σ_w^2, respectively. The variance of the generalized noise model is

$$\sigma^2 = f^{2\gamma} \cdot \sigma_u^2 + \sigma_w^2. \tag{2}$$

By changing the three NLF parameters γ, σ_u^2, σ_w^2, this generalized noise model can represent various types of noise such as film-grain noise, multiplicative speckle noise, Poission noise and others [8].

This generalized noise model is often used in the SDN identification [4, 5, 6, 7]. The SDN identification with this-general noise model is to estimate the three NLF parameters.

Existing SDN identification algorithms [4, 5, 6, 7] usually assume that one parameter is known and thus only estimate the remaining two parameters to simplify the problem. In contrast, the proposed SDN identification algorithm simultaneously estimate all the three parameters.

2.2. Patch-based approach

In our previous work [2], we have proposed the patch-based noise level estimation algorithm for the signal independent noise. For the SDN identification, we also apply the patch-based approach. Firstly we extract the weak textured patches. Then, using the weak textured patches, we estimate the three NLF parameters assuming that the variance of noise in the patch is constant. We need the noise-free pixel values and the noise variance of the patches to estimate the three NLF parameters by the Maximum-Likelihood estimator. We approximate the noise-free pixel values of the patches by the mean values, assuming that the patches are flat with zero-mean noise. We follow the same manner as in [2] for the noise variance estimation. The noise variance of the patches are estimated by the power of noisy patch along to the eigenvector associated to the minimum eigenvalue. The estimation of the noise-free signal and the noise variance are:

$$\begin{aligned}\hat{f} &= \frac{1}{N}\sum_{j=1}^{N} g_j \\ \hat{\sigma}^2 &= \|\mathbf{u}_{min}^T \cdot \mathbf{g}\|^2,\end{aligned} \tag{3}$$

where $\hat{f}$ is the estimation of the noise-free signal, g_j is the j-th pixel value in the observed patch, N is the number of pixels in the observed patch, $\hat{\sigma}^2$ is the estimation of the noise variance, $\mathbf{g}$ is the vector representation of the observed patch, $\mathbf{u}_{min}$ is the minimum eigenvector of the covariance matrix of the extracted weak textured patches and T is the transpose operator. We also apply an iterative framework same as in [2]. In the following section, we describe the weak textured patch selection and the ML estimator.

2.3. Weak textured patch selection

We have proposed a texture strength metric and a weak textured patches selection algorithm based on the texture strength. We follow the same manner. The texture strength is

$$\xi = tr(\mathbf{G}\mathbf{G}^T), \tag{4}$$

where $tr(\cdot)$ is the trace of the matrix, and $\mathbf{G}$ is the gradient matrix defined by

$$\mathbf{G} = \begin{bmatrix}\mathbf{D}_u\mathbf{g} & \mathbf{D}_v\mathbf{g}\end{bmatrix}, \tag{5}$$

where $\mathbf{D}_u$ and $\mathbf{D}_v$ are the matrices which represent the horizontal and vertical derivative operator, respectively. The weak textured patches are extracted by thresholding the texture strength. The threshold has been derived based on the statistical analysis as

$$\delta = F^{-1}(\tau, \frac{N}{2}, \frac{2}{N}\sigma^2 tr(\mathbf{D}_u^T\mathbf{D}_u + \mathbf{D}_v^T\mathbf{D}_v)), \tag{6}$$

where δ is the threshold of the texture strength, $F^{-1}(\tau, \alpha, \beta)$ represents the inverse gamma cumulative function with shape parameter α and scale parameter β, τ is the confidence level, and σ^2 is the noise variance of the patch. The threshold in Eq.(6) is a function of the noise variance. The confidence level is empirically set. In this paper, we use 0.99 for the confidence level. The mean pixel value and noise variance are firstly estimated according to the model in Eq.(3). Then the threshold of each patch is determined by Eq. (6). As mentioned here, the noise variance is required to determine the threshold. Therefore, we apply the same iterative framework as in [2].

2.4. Maximum likelihood estimation

Using the selected weak textured patches, we identify the three SDN parameters with estimations of noise-free pixel values and the noise variance by the ML estimator. The likelihood with selected weak textured patches is

$$\mathcal{L} = \prod_{k=1}^{M} \frac{1}{\sqrt{2\pi\sigma^2(\hat{f}_k;\gamma,\sigma_u^2,\sigma_w^2)}} \exp\left\{-\frac{\hat{\sigma}_k^2}{2\sigma^2(\hat{f}_k;\gamma,\sigma_u^2,\sigma_w^2)}\right\}, \tag{7}$$

where M is the number of selected weak textured patches, $\hat{f}_k$ is the mean pixel value of the k-th patch, $\hat{\sigma}_k^2$ is the estimated noise variance of the k-th patch. The cost function to be minimized can be derived from negative log-likelihood function as

$$E(\gamma,\sigma_u^2,\sigma_w^2) = \sum_{k=1}^{M}\left[\log\sigma^2(\hat{f}_k;\gamma,\sigma_u^2,\sigma_w^2) + \frac{\hat{\sigma}_k^2}{\sigma^2(\hat{f}_k;\gamma,\sigma_u^2,\sigma_w^2)}\right]. \tag{8}$$

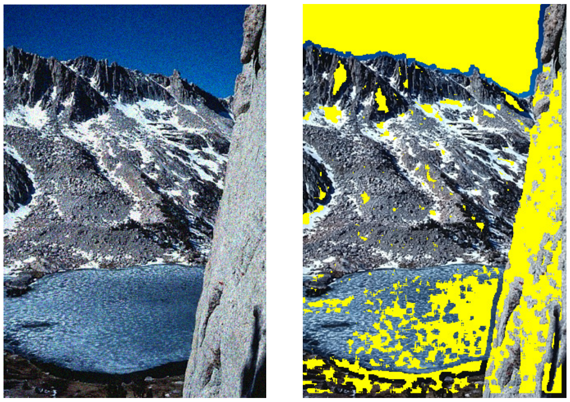
We apply the gradient-descent algorithm to minimize the cost function with respect to the three parameters $\gamma, \sigma_u^2, \sigma_w^2$. The gradient-descent algorithm requires the initial guess. In this paper, we simply set zeros for three parameters.

3. EXPERIMENTAL RESULTS

3.1. Results on noise parameter estimation

The proposed algorithm[1] is assessed by using synthesized noisy images. We use natural images from BSD dataset [9] and generate synthetic signal dependent noise according to

[1] MATLAB code is available on the author's webpage. `http://www.ok.ctrl.titech.ac.jp/res/NLE`



(a) Noisy image ($\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 5$) (b) Selected weak textured patches

Fig. 1. Results of weak textured patches selection for *mountain* image.

the general noise model in Eq. (1). Several noise patterns are synthesized by changing the three SDN parameters. The mean value and standard deviation of each estimated parameter are evaluated. For each pixel intensity in the range of $[0, 255]$, the RMSE of estimated noise level,

$$RMSE = \sqrt{\frac{1}{256}\sum_{i=0}^{255}\|\sigma^2(i;\hat{\gamma},\hat{\sigma}_u^2,\hat{\sigma}_w^2) - \sigma^2_{true,i}\|^2} \quad (9)$$

is also calculated to evaluate the accuracy of the algorithm. We compare the proposed algorithm with Torricelli's algorithm [5] and Zabrodina's algorithm [7]. Note that the parameter σ_w^2 has to be known in Torricelli's algorithm [5], and the parameter γ has to be known in Zabrodina's algorithm [7], while the proposed algorithm can simultaneously estimate all three parameters. The Fig.1 (a) shows an example of synthetic noisy image and the yellow part in Fig.1 (b) is the selected weak textured patches by the proposed algorithms. We can see that most weak textured patches are correctly selected by the proposed algorithm. The NLFs estimated by different algorithms and the ground truth are show in Fig.2. We can find that the NLF estimated by the proposed algorithm is closer to the ground truth compared with the existing algorithms.

The statistic of estimated noise parameters and RMSE of noise level functions tested on BSD image dataset are shown in Table 1. For various setting noise parameters, the proposed algorithm estimates the noise parameter and noise level more accurately than the existing algorithms.

3.2. Results on image denoising

The accurate SDN estimation can improve the performance of denoising. Although there are not so many researches on the denoising filter for SDN, the adaptive Wiener filter [10] is a well-known one. The Wiener filter can be

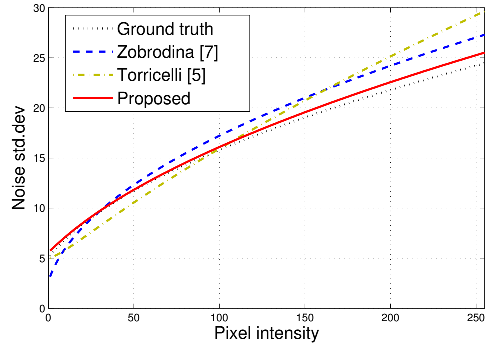


Fig. 2. Noise level estimation results for *mountain* image. ($\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 5$)

expressed by:

$$\hat{f} = \mu + \frac{\nu^2 - \sigma^2}{\nu^2}(g - \mu) \quad (10)$$

where μ and ν^2 are local mean and local variance of the neighborhood and g is the observed pixel value. In theory, the parameter σ^2 should be the local noise variance. Therefore we can use the SDN estimation results. In some implementation like MATLAB default setting, the parameter σ^2 is set to be the average of all local variances. In this paper, we call it the simple adaptive Wiener filter. We compare the simple adaptive Wiener filter and the adaptive Wiener filter with three different SDN estimation algorithms: the proposed algorithm, Torricelli's algorithm [5], and Zabrodina's algorithm [7]. Table 2 shows the average PSNRs on BSD dataset with different SDN parameters. We can see the better denoising performance when using our SDN estimation.

4. CONCLUSION

In this paper, we have proposed an algorithm to estimate the noise parameters of the generalized signal dependent noise (SDN) model from a single noisy image. The noise parameters are estimated based on correctly selecting of the weak textured patches. The advantage of the proposed method is that: it does not require any prior knowledge of the noise type. The experiments on synthetic noisy image data have shown that the proposed algorithm outperforms the existing algorithms. And using our noise level estimation, the denoising performance of the adaptive Wiener filter can be effectively improved.

5. REFERENCES

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Table 1. SDN parameter estimation of different algorithms on BSD dataset

Parameters	Methods	γ		σ_u		σ_w		RMSE of noise level
		Avg.	Std.Dev	Avg.	Std.Dev	Avg	Std.Dev	
$\gamma = 0.5$, $\sigma_u = 0.5$, $\sigma_w = 5$	Torricelli [5] (σ_w known)	0.67	0.36	0.92	2.34	-	-	55.16
	Zabrodina [7] (γ known)	-	-	0.64	0.41	5.69	2.69	65.69
	Proposed	0.49	0.23	1.06	1.02	4.09	2.03	**8.53**
$\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 5$	Torricelli [5] (σ_w known)	0.57	0.12	1.14	0.72	-	-	81.98
	Zabrodina [7] (γ known)	-	-	1.57	0.40	5.93	2.94	88.75
	Proposed	0.50	0.10	1.66	0.63	4.04	3.03	**16.72**
$\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 15$	Torricelli [5] (σ_w known)	0.734	0.295	0.704	0.842	-	-	149.63
	Zabrodina [7] (γ known)	-	-	1.50	0.16	15.04	1.12	41.90
	Proposed	0.49	0.15	2.24	1.80	13.67	2.86	**23.94**
$\gamma = 0.5$, $\sigma_u = 2.5$, $\sigma_w = 5$	Torricelli [5] (σ_w known)	0.54	0.07	1.93	0.83	-	-	195.80
	Zabrodina [7] (γ known)	-	-	2.46	0.58	6.91	3.20	406.57
	Proposed	0.49	0.05	2.67	0.65	3.76	4.06	**31.74**
$\gamma = 0.7$, $\sigma_u = 0.5$, $\sigma_w = 5$	Torricelli [5] (σ_w known)	0.73	0.06	1.17	0.37	-	-	722.30
	Zabrodina [7] (γ known)	-	-	10.47	4.04	2.78	6.71	148965.70
	Proposed	0.67	0.64	1.77	0.42	3.89	4.25	**17.81**

Table 2. Denoising performance (PSNR) of adaptive Wiener filter with different SDN parameters on BSD dataset

Parameters	Simple adaptive Wiener filter	Torricelli [5] (with σ_w known)	Zabrodina [7] (with γ known)	Proposed method
$\gamma = 0.5$, $\sigma_u = 0.5$, $\sigma_w = 5$	29.025	33.258	33.175	**33.426**
$\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 5$	27.399	27.932	28.213	**28.302**
$\gamma = 0.5$, $\sigma_u = 1.5$, $\sigma_w = 15$	26.552	25.979	**26.804**	26.731
$\gamma = 0.5$, $\sigma_u = 2.5$, $\sigma_w = 5$	25.416	24.786	25.474	**25.719**
$\gamma = 0.7$, $\sigma_u = 0.5$, $\sigma_w = 5$	22.261	21.517	23.136	**28.973**

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